

Mathematical Mayhem

Symbolism, Symmetry, and Spirals in *Vertigo*

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ABSTRACT

While mathematics and film studies are often seen as disparate areas, they can intersect and enhance each other. In *Vertigo* (1958), Alfred Hitchcock uses the golden ratio and golden spiral to juxtapose the desire for order within the chaotic moments of Scottie's (James Stewart) investigation into Madeleine's (Kim Novak) life. The spiral motif directs viewers to look closer at various aspects of both the mise-en-scène and the characters while inversely understanding less about each of them as the film progresses. The golden ratio connects to key moments of the film and highlights Hitchcock's desire to invite the viewer into Scottie's world while sending them down a spiralling path of chaos, confusion, and deception. Hitchcock further utilizes mathematics through his use of symmetry and reflection, accentuating the similarities and differences amongst characters, themes, and locations. This article will thus explore how understanding the mathematical symbolism in *Vertigo* can allow for a deeper affinity and appreciation of Hitchcock's masterpiece.

Alfred Hitchcock, known as the Master of Suspense, applied any and every technique at his disposal to deepen and intensify his films, even unconventional techniques associated with mathematics, a discipline which is not commonly associated with film production but which, in reality, is the unifying thread that runs through all of life. Mario Livio, head of the Science Division at the Space Telescope Science Institute, notes how "numbers and mathematics have the curious propensity of contributing even to the understanding of things that are, or at least appear to be, extremely remote from science" (1). For example, mathematics dictates the arrangement of leaves climbing up the stem of a plant and how their arrangement maximizes the surface area for the absorption of sunlight; mathematics explains beauty from

a quantitative standpoint, highlighting why the audience's eyes are immediately drawn to specific characters, items, or settings; and mathematics clarifies and intensifies the growing tension and suspense in Hitchcock's *Vertigo*.

Hitchcock once stated, "I find that the easiest way to worry people is to turn the tables on them" (Gottlieb 123). One of the ways he achieved this was to use mathematics as a tool to help keep people in a constant state of suspense. The opening credits of *Vertigo* introduce the audience to Hitchcock's chaotic world. By fracturing the woman's face within the shots, Hitchcock hints at how the coming story will unfold; the full story will not be visible or easily understood. Only parts and pieces of the tale will be unveiled at various moments along the way, so the audience



Fig. 1 | Title in Hitchcock's *Vertigo*, 00:01:19. Alfred J. Hitchcock Productions, 1958.



Fig. 2 | Spiral emerging from eye in Hitchcock's *Vertigo*, 00:01:25. Alfred J. Hitchcock Productions, 1958.

is continually anticipating the unknown and building suspense around what little they think they do know. The closeness of these opening shots adds intimacy and discomfort to the film. The audience is immediately thrust into a situation where they are up close and personal with someone on the screen. The social decorum of personal space has been violated. Thus, the audience is prepped to look closely at the characters as their stories unfold.

Tension and unease continue to build as the title *Vertigo* grows out of a woman's eye (Fig. 1). Seeing one eye centred and isolated within the frame is unsettling. Film critic Michael Howarth comments on how "a shot of an eye symbolizes a new angle or perspective; it alerts the viewer that he or she (or even one of the characters) will soon change his or her perspective and begin to see the world in a new light" (198). Perspectives will constantly be changing and evolving throughout *Vertigo* as new evidence is brought to light. Even the title *Vertigo* suggests Hitchcock's desire to cause unsettling, destabilizing sensations in his audience as they view the film. Then, as the colour shifts from a muted palate to unnatural red in the opening title sequence, Hitchcock alludes to the impending danger the woman presents. In this film, Hitchcock frames women as untrustworthy. By scrutinizing the female characters, the audience will experience their own vertiginous sensations. The assumed facts they are holding tightly will disintegrate within their hands as the ground falls away with each new revelation.

Next, the spiral appears in the woman's eye behind the title (Fig. 2). Hitchcock uses the spiral to draw the audience into a

reality that is full of duplicity and chaos, a reality that leaves them wondering what is real and what is not. Patrick McGillian points to the math behind the opening sequence and how "Saul Bass... put his unique stamp on the title sequence...with a distinctive prologue that juxtaposed images of eyes with dizzying Lissajous spirals" (561). Spirals are mathematical representations of different types of equations. An important characteristic about a spiral is its movement around a centrally fixed point, known as the centre. Spirals come in many differing forms: logarithmic, hyperbolic, Archimedean, and golden, just to name a few. Hitchcock uses women, specifically Madeleine (Kim Novak), as the centre around which Scottie (James Stewart), and consequently the audience, spirals. Madeleine, as the centre of both Hitchcock's story and Scottie's obsession, becomes the focus for the audience and the origin of chaos and confusion. The placement of the spiral within the woman's eye following the title primes the audience to question the validity and trustworthiness of the women and to expect a spiralling chaotic world in the film to follow. Therefore, the spiral is a perfect symbol for *Vertigo* because the more closely one follows Hitchcock's spiralling story in search of the truth, the further away from the truth one actually becomes.

Hitchcock hides the golden spiral (a logarithmic spiral with a growth factor of the golden ratio, approximately 1.618...) throughout the film. Stephen Ornes, an award-winning writer and scholar on science and mathematics, observes how "the golden ratio...often invokes ideas of harmony and elegance" (34). Hitchcock thrived on turning the audience's expectations on their heads, and he commented that "suspense involves contrast" (Gottlieb 115). The golden ratio is used throughout *Vertigo* to highlight moments of discovery but also moments where fractured stories are exposed. Hitchcock, instead of using the golden spiral to harmonize, uses the golden spiral to add layers of confusion and deception.

Vertigo opens with a rooftop chase scene. Scottie stumbles during the chase and hangs from the side of the building while a fellow officer attempts to help him. This scene highlights the disorienting change of perspective experienced by people suffering from vertigo. As Scottie dangles from the rain gutter, the ground rapidly falls away. He is frozen in fear, and his fellow partner (Fred Graham) falls tragically to his death. The medical condition of vertigo is only mentioned here in detail at the beginning of the film. By giving his protagonist a limitation, Hitchcock makes Scottie more human and more relatable to the audience. Due to this vertigo, Scottie quits the police force. *Vertigo* forces Scottie to shift his perspective and identity. It is Scottie's vertigo that sets off the chain of events and leads him straight to Gavin Elster (Tom Helmore) and his wife, Madeleine.

As Scottie enters his final days of recovery following the tragic accident on the rooftop, he discusses the upcoming reunion with his old college chum, Gavin, with his longtime friend and previous lover, Midge (Barbara Bel Geddes). Hitchcock utilizes a monochromatic colour palate in Midge's apartment to hint at Midge's personality and her relationship to Scottie. She is shown in a yellow blouse while Scottie lounges in her yellow apartment. The colour yellow is chosen to highlight both Midge's creativity (she is an artist and undergarment



Fig. 3 | Hidden golden spiral in Midge's apartment in Hitchcock's *Vertigo*, 00:09:30. Alfred J. Hitchcock Productions, 1958.

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designer) and her cowardice. Midge and Scottie were once engaged, but she broke off the relationship. Midge is afraid of her feelings for Scottie and harbors regret for having broken off their relationship in the past. Her regret is further evidenced later in the film when she paints herself into a replica of Carlotta's portrait, trying to win over Scottie's attention and affections.

Hitchcock also uses yellow in association with Midge because she is Scottie's processor, the one he turns to in order to work through his thoughts and questions surrounding the mysterious Madeleine. Leatrice Eiseman, executive director of the Pantone Colour Institute, notes how yellow "is believed to stimulate the left side of the brain to prepare for more logical thinking, clearing the mind in order to provide the clarity for reasonable decision making" (38). Midge's attire and apartment being yellow provide the clarity and stimulation Scottie needs to try and address his vertigo, confront his confusion about Madeleine, and help prepare him for the twisted tale unraveling before him.

The first emergence of the golden spiral, outside of the title sequence, can be seen as Scottie discusses his acrophobia with Midge in her apartment (Fig. 3). Midge is the centre around which the spiral originates as she discloses to Scottie how unlikely it is that he will ever overcome his fear of heights

without another shockingly traumatic situation. Midge is the giver of knowledge in this moment, but the spiral also showcases Midge's focus on her work and not Scottie. Scottie stands to the side of the frame, away from Midge and her work, signifying his removal from her attention and life over the years. Additionally, Scottie's distance from the centre of the spiral calls attention to his lack of control over his fear.

Hitchcock employs another mathematical concept in the next scene: symmetry. Symmetry is the "intrinsic property of a mathematical object which causes it to remain invariant under certain classes of transformations" (Weisstein). As Scottie visits Gavin Elster, he learns of Elster's desire to have his wife followed. Elster believes his wife may be possessed by her late great-grandmother, Carlotta. Scottie does not believe in the supernatural and does not want to be a private investigator for Elster. As writer and critic Paul Duncan observes, "when Scottie meets Elster, the framing is symmetrical, their body language is the same then changes as they talk, Scottie starting high (in power position) and Elster low, and ending up reversed as Scottie is persuaded by Elster's argument" (131-132). While subtle, the arrangement of Scottie and Elster throughout this interaction shows the shifting of power between the two men and the shifting from equality and symmetry to an imbalanced dissimilarity; this change highlights how "Geometry is a journey—that roughness inhabits the curves of reality, that beauty and symmetry can take complicated forms" (Ornes 69). The symmetry between the men shifts during this scene as Scottie surrenders control to Elster, who will then use that power to manipulate Scottie into believing the tumultuous tale to come.

Although he still doubts the validity of Elster's concerns, Scottie agrees to visit Ernie's that night in order to catch a glimpse of Madeleine. Her reveal comes with many layers, masterfully assembled by Hitchcock. Madeleine, as she walks to the bar preparing to leave Ernie's, pauses just behind Scottie. This close-up shot of Madeleine's profile implies a duality to Madeleine's character. Scottie is not privy to a full shot of Madeleine's face at this time, suggesting hidden aspects of her character. The bright red wallpaper within the restaurant adds a layer of symbolism to the shot as well. Red typically signifies danger and, contrastingly, love, both of which will be experienced by Scottie during his pursuit of Madeleine and the truth.

Colour theory has been utilized by artists for millennia, but only since the invention of colour television and movies has it begun to play a role in cinema. While colour theory is typically associated with art and design, it intersects with mathematics as well. Engineer Ethan Gardner explains how math supports the understanding of complementary colours and analogous colours in his article "A Math-Based Approach to Colour Theory":

A complementary colour scheme is a two-colour combination consisting of a base colour (H_0) and another colour (H_1) that is 180 degrees apart from H_0 on the colour wheel...Analogous colour schemes use a combination consisting of a base colour (H_0) and one or more adjacent colours (30 degrees apart) on the colour wheel.

Mathematics helps underline the relationship between the colours and how they interact with each other, allowing for richer symbolic meanings.

When Madeleine is first viewed, she is wearing a stunning green shawl that stands out against the vibrant red interior of Ernie's. The choice to put Madeleine in green produces many implications. Firstly, the green in Madeleine's shawl complements the red of the restaurant's walls; red and green are located 180 degrees apart on the colour wheel. It is important to note that complementary colours "enhance or emphasize the qualities of their opposites" and when they are "placed immediately next to each other, they appear more intense...each complement intensifies the brilliance of the other and seems to visually vibrate" (Eiseman 20). Madeleine stands out amongst the crowd because her outfit dramatically contrasts with her surroundings. Secondly, the green in Madeleine's dress is symbolic. Green symbolizes renewal and regrowth. Madeleine will continue to live on and be reborn through Carlotta and Judy as the story unfolds. Thirdly, green acts as a physical representation of temptation, a point echoed by cultural historian Kassia St. Clair when she remarks how in *The Wizard of Oz* the Emerald City "is a metaphor for the magical fulfillment of dreams: it lures the characters in because they all want something from it" (220). Madeleine is the personification of the Emerald City for Scottie, bringing him into her convoluted story through intrigue. Scottie desires to know her story, why she is acting strangely, and along the way, she becomes the fulfillment of his dreams. Scottie cannot restrain his own emotions, and he forms emotional connections with Madeleine that blind him to her deception and dangerousness.

After initially viewing Madeleine at Ernie's, Scottie decides to follow her around for a day to see if he can discover any clues to her seemingly odd behaviour. He observes Madeleine buying flowers at a flower shop, visiting Carlotta Valdez's grave, staring transfixed at a portrait of Carlotta in a museum, and visiting the McKittrick Hotel, Carlotta's old home. Carlotta is the common denominator that seems to be guiding Madeleine and her actions. Through Scottie's surveillance, the audience is introduced to another mathematical concept: reflection.

In mathematics, reflections can be examined with respect to functions. Typically, these functions are studied in the most rudimentary sense by discussing reflections across the x and y axis in the Cartesian coordinate system. The whole concept of reflection looks at how the original can be transformed to mirror itself across a dividing line. Madeleine is a reflection of Carlotta across the line of time. Her daily activities, as witnessed by Scottie and subsequently the audience, are all closely tied to Carlotta and her life. Reflections will continue to play an integral role throughout the film as Madeleine reflects Carlotta and then Judy is forced, by Scottie, to reflect Madeleine. Hitchcock uses the mathematical concept of reflection to insinuate commonalities between Carlotta and Madeleine, allowing for conclusions to be drawn about Madeleine's impending fate.

Hitchcock also uses the golden spiral in Scottie's observations of Madeleine to highlight various connections between the characters. While viewing Madeleine's behaviour at the art



Fig. 4 | Spiral connection between Madeleine and Carlotta in Hitchcock's *Vertigo*, 00:26:35. Alfred J. Hitchcock Productions, 1958.



Fig. 5 | Spiral connection between Madeleine and Carlotta in Hitchcock's *Vertigo*, 00:26:42. Alfred J. Hitchcock Productions, 1958.

The golden ratio is used throughout *Vertigo* to highlight moments of discovery but also moments where fractured stories are exposed.

museum, the camera zooms in on the flowers she has brought before slowly panning up to reveal that they are the same flowers Carlotta is holding in her portrait, yet another example of reflection. The next close-up shot is on the spiral updo of Madeleine's hair (Fig. 4). Again, the camera carries the viewer's eye to the portrait of Carlotta, while deliberately closing in on the same spiral that is present in Carlotta's hair (Fig. 5). Hitchcock uses the physical spiral within the women's hair to act as a unifier between the two women, signaling connectedness and reflection, but it also hints at Madeleine's spiralling mental instability. It is feared that she is wandering down the same dark, spiralling path as Carlotta. If the reflections between the two women do not cease, then Madeleine's life will end in tragedy.

The golden spiral will continue to unite the many transformations of Carlotta through Madeleine and eventually Judy (Kim Novak). In mathematics, as in nature, the spiral suggests consistency, predictability, and beauty. Mathematics is the one constant throughout all of time, but Hitchcock likes to take one's expectations and corrupt them. If the audience is expecting predictability when they view the spiral, then Hitchcock is

going to connect the spiral to chaos and disorder. Each time the spiral reappears, Madeleine and Scottie have traveled further down the rabbit hole into a world that seems normal but is anything but.

Following Scottie's surveillance of Madeleine, Hitchcock hides the golden spiral within the bookshop scene. Scottie has once again turned to Midge for help in processing what he has witnessed and is led to Argosy's Bookshop. Hitchcock arranges the characters within this scene by the golden spiral, showcasing their connectedness to each other and the story being told (Fig. 6). Within this scene, Midge is closer to the bookshop owner as the story is told; she connects Scottie to this new information, so Hitchcock places Scottie further away from the source of information physically. Hitchcock, here, directs the audience to pay attention, to draw close to these new facts, but he also suggests that by focusing on Carlotta's story, one will be sent spiralling into the unknown.

Pop Leibel (Konstantin Shayne), the proprietor of Argosy's Bookshop, reveals many dualities present within Carlotta's sad tale. In mathematics, duality fundamentally "gives two different points of view of looking at the same object" (Atiyah 69). Pop Leibel explains various dualities within Carlotta herself, which then get projected onto Madeleine. He refers to Carlotta as "Beautiful Carlotta. Sad Carlotta" (00:34:15). Her story in and of itself has duality. Her beauty made her desirable, but when she became obsolete to the rich man she loved and he disposed of her, she transformed into the "Sad Carlotta" Pop mentions. This story mirrors Elster and Madeleine's story. The entire latent plot of the film spirals around Elster's desire to rid himself of his wife and the convoluted way he goes about achieving that. It is important to note that because of the reflection and symmetry between Carlotta and Madeleine, Hitchcock is providing the audience with foreknowledge of what to expect when it comes to Madeleine and her fate.

Madeleine unveils the duality that exists between herself and Carlotta when she takes Scottie to the Sequoia National Park. During this scene, Madeleine's hair is once again coiffed up in a spiral, reflecting Carlotta's presence and drawing attention to their duality. Scottie asks Madeleine if she has ever visited the forest before, which she denies. Hitchcock presents mounting evidence of Madeleine's possible possession by Carlotta such as the spiral as the symbol for Carlotta's presence within Madeleine.

Hitchcock uses Scottie's knowledge about the trees to bring to light duality and reflection amongst Madeleine and Carlotta and, subsequently, Judy. Scottie tells Madeleine about the trees, that "their true name is *sequoia sempervirens*—always green, ever living" (00:58:45). This quotation connects directly to Hitchcock's use of the colour green in reference to Madeleine, and this visual imagery also emphasizes the duality present between Carlotta and Madeleine. Paul Duncan describes how the name of the trees "explains why, when we first see Madeleine, and then later Judy Barton, she is wearing green" and how "the green light (from the neon sign) bathes Judy when she is transformed back into Madeleine. Carlotta becomes Madeleine becomes Judy—they are objects of love reincarnated, remade, over and over. They are everlasting...It's a romantic,

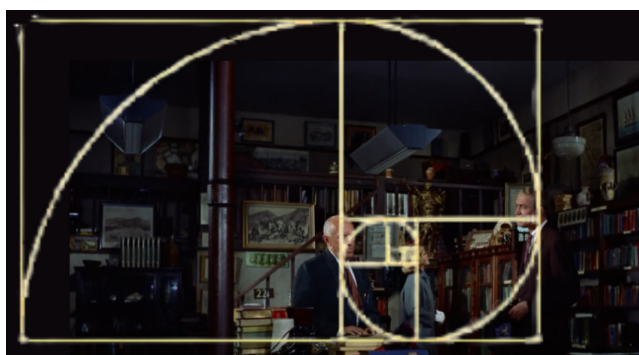


Fig. 6 | Hidden golden spiral in Argosy's Bookshop in Hitchcock's *Vertigo*, 00:34:15. Alfred J. Hitchcock Productions, 1958.



Fig. 7 | *Vertigo* staircase in Hitchcock's *Vertigo*, 01:16:34. Alfred J. Hitchcock Productions, 1958.

gothic idea played out in bright sunshine" (132). The trees act as a symbolic reflection of Carlotta's story upon Madeleine.

Hitchcock again utilizes the spiral motif within the forest through the cross-section of a tree. Madeleine points at a ring within the tree and says, "Here I was born, and here I died" (00:59:41). Following this declaration, Madeleine walks away into the forest like a spectral being fading from existence and does not respond to Scottie calling out her name. The spirals within her hair and in the tree stump where she located her declared birth and death dates point to Carlotta's apparent full possession of Madeleine, at least for fleeting moments. Hitchcock has succeeded in using the spiral symbolism to heighten the tension and suspense within *Vertigo*. All of these spiralling connections, reflections, and dualities point directly to the suspenseful crescendo that will crash down upon the characters.

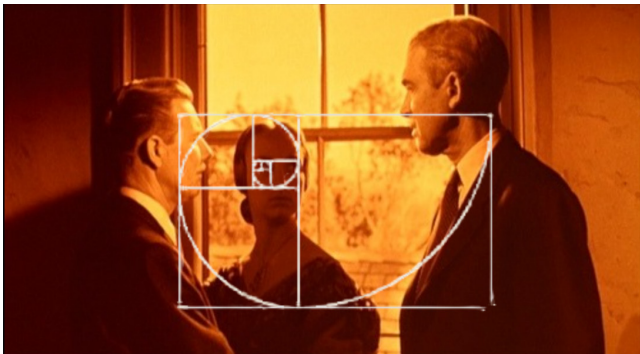


Fig. 8 | Hidden golden spiral in Scottie's dream in Hitchcock's *Vertigo*, 01:24:27. Alfred J. Hitchcock Productions, 1958.

Hitchcock likes to use parallelism—consequently, another mathematical concept—to reiterate important themes and ideas throughout *Vertigo*.

The battle for Madeleine and her life culminates at the Spanish Mission south of San Francisco. Scottie takes Madeleine there in hopes of reconnecting her to reality, of helping her overcome her unknown fears, and to break the connection that seems to have formed between her and Carlotta. Madeleine, hair upswept into spirals for one last time, seems dazed. She flits between the present as herself and the past as Carlotta, highlighting the existing duality and reflection between the women. Finally, it appears Carlotta has overtaken Madeleine entirely, and she rushes for the church bell tower. Scottie, determined to save Madeleine, and redeem himself for failing to save his fellow officer at the beginning of the film, follows and stumbles into one of Hitchcock's most iconic scenes: the vertigo staircase.

As Madeleine, presumably under Carlotta's possession, races up the stairs with Scottie in pursuit, his vertigo is triggered, and the audience actually gets to experience the golden spiral and its dizzying sensation (Fig. 7). Spiralling up and around, trying to catch Madeleine, Scottie glances over the banister and is caught in a cold sweat as the floor quickly falls away from him, inducing the spiralling, chaotic feelings of uneasiness, imbalance, and uncertainty. Hitchcock admitted to Francois Truffaut that he had "thought about the problem [of the spiralling camera

movement] for fifteen years. By the time we got to *Vertigo*, we solved it by using the dolly and zoom simultaneously" (246). Mastering this complex camera motion allowed Hitchcock to impose vertigo on the audience while witnessing Scottie's paralysis in his pursuit of Madeleine. These stairs, which personify the golden spiral, symbolize Scottie's loss of control of the investigation, of his own body, and of Madeleine. Everything has spiraled out of order.

Madeleine's death is the suspenseful crescendo Alfred Hitchcock set out to produce, but without the symbolic and literal spiralling staircase, much would have been lost. Madeleine's death also signals another duality within the film: two people have now perished due to Scottie's inability to overcome his vertigo. Reflection accentuates the falling death of Madeleine by reminding the audience of the officer's fall from the rooftop. Hitchcock likes to use parallelism—consequently, another mathematical concept—to reiterate important themes and ideas throughout *Vertigo*.

Following Madeleine's death, Scottie's life spirals into darkness. He sits through a trial to determine if he is liable for her death. Elster and Scottie chat by the window following the reading of the verdict. Elster is leaving San Francisco and Scottie is lost in his grief. This scene will shortly be reflected within Scottie's dream, highlighting some unknown truth integral to the overall plot of the film.

Hitchcock employs a twisted, confusing dream sequence with absurd colours and animation to showcase Scottie's grief-stricken subconscious and to offer up clues to the mystery of Madeleine. Scottie's head is isolated in the centre of the frame in the opening scene of the dream sequence, indicating his isolation and loneliness following Madeleine's death. The strobing effect of the colour transitions illustrates the tumultuous shift happening within Scottie and his mental state. The grief and loss are overwhelming him, colouring his world and all that he sees and experiences.

An artistic rendering of Madeleine's flowers—which incidentally are the same flowers Carlotta holds in her portrait, yet another example of duality—fills the next shot. The flowers morph into ambiguous coloured shapes, exploding outward towards the screen. This imagery represents both the duality in Madeleine and Carlotta, as both women possessed the same flowers, but it also signifies how their stories implode from within. Both women are pushed to the brink emotionally and mentally and resort to suicide. The exploding flowers represent how their lives became overwhelming and ended in their self-destruction.

The dream continues to offer up more vital information regarding Madeleine's story. Mirroring the scene only moments before, Scottie is shown standing at the window with Elster, only this time Carlotta (Joanne Genthon) stands between them. Another golden spiral is hidden within this shot in the arrangement of the characters, with the focus of the spiral being Carlotta (Fig. 8). Elster's hands on Carlotta notify viewers of his control over her story and how it was portrayed to Scottie. Elster manipulated Carlotta's story to fit the twisted tale he sold as Madeleine's. This scene is integral to understanding the deception being perpetrated by Elster. Carlotta is gazing

intently up at Scottie, begging him with her eyes to realize the scheme that's been playing out. Hitchcock gives just enough for the audience to begin working out the duplicitous nature of Elster and Madeleine.

Scottie, sadly, ends up in a mental hospital while he tries to come to grips with failing, yet again, to protect someone on his watch and begins to process his loss and grief. Midge is shown in blue while she attempts to awaken Scottie from the depths of his grief. Her apparel is blue in the hospital, symbolizing the calming demeanor she strives to portray to Scottie during his loss. The blue of her outfit also alludes to her sadness at watching the man she loves fall apart emotionally before her. Midge's outfit colours—yellow at the beginning of the film and blue following Madeleine's death—point to her role in Scottie's life. As previously mentioned, complementary colours sit 180 degrees apart on the colour wheel. Yellow and blue, the two colours most associated with Midge throughout *Vertigo*, are complementary colours on both the RGB (Red Green Blue) and CMY (Cyan Magenta Yellow) colour wheels. This is significant because “as the name suggests (complementary and not complimentary), these colours help each other stand out. They bring out the best in each other by making their complement more vibrant or noticeable” (IxDF). Midge's role has been as a friend, a processor, and a caregiver. She helps bring out the best in Scottie throughout the film, and Hitchcock's use of colours communicate and magnify different layers to their relationship.

In the next sequence, the slow panning shot of San Francisco marks the passage of time, informing the audience that Scottie has spent much time recovering from Madeleine's untimely demise. Hitchcock expertly uses *mise-en-scène* to alert viewers to his multilayered masterpiece. As Scottie stands mournfully outside of Madeleine's old residence, her green car—note the symbolism of renewal and rebirth again—appears on the left-hand side of the frame. Near the centre of the shot is a “One Way” sign facing Madeleine's old car. Here Hitchcock is inviting viewers in on an ironic little secret: there's always more than one way to interpret a story.

While coming to terms with Madeleine's death, Scottie revisits all her old haunts: Ernie's, Carlotta's portrait at the museum, and the flower shop. While visiting the flower shop, Scottie encounters a reflection of Madeleine. This scene introduces Scottie and the audience to a new transformation of Madeleine, and subsequently Carlotta: Judy Barton. Scottie's first view of Judy mirrors that of his first observation of Madeleine—both women are clothed in green. Hitchcock is reiterating the eternal aspect of these women through the use of this colour. While Madeleine's dress is shrouded in a green shawl, Judy is clad entirely in green. Additionally, the shot of Judy talking to her girlfriends on the street is flanked by green cars, stressing the rebirth motif. Carlotta was reborn into Madeleine who is reborn as Judy, creating reflection after reflection after reflection.

Unbeknownst to Scottie, Judy is not just a reflection of Madeleine; Judy is Madeleine. After forcibly inviting Judy to dinner, the audience is privy to Judy's inner thoughts through a sequence of flashbacks uncovering the truth of what happened



Fig. 9 | Madeleine's profile in Hitchcock's *Vertigo*, 00:18:05. Alfred J. Hitchcock Productions, 1958.

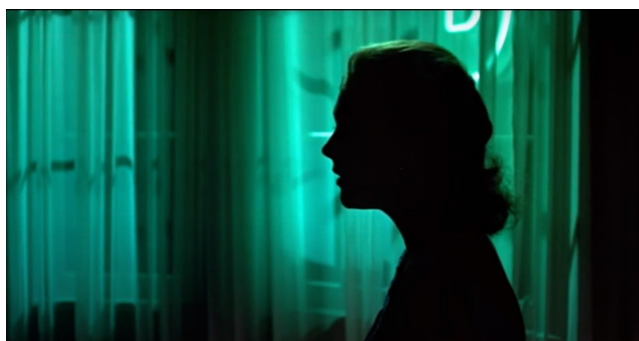


Fig. 10 | Judy's profile in Hitchcock's *Vertigo*, 01:44:40. Alfred J. Hitchcock Productions, 1958.

at the Spanish Mission. As Judy pens a letter to Scottie confessing the whole tragic tangled tale, the camera slowly spirals around her, mimicking the golden spiral. Instead of adding layers of deception and increasing the chaotic loss of control, the truth is finally being revealed and unraveled before the audience. Elster, having discovered Scottie's limitations from the newspaper, used Judy to play Madeleine, arranging a staged scene so he could kill his wife. Scottie has spent the entirety of the film following spiralling stories of betrayal and disinformation. While Scottie is still in the dark on who Judy really is, Hitchcock has ratcheted up the suspense as the viewer anxiously awaits how this new truth will affect the characters onscreen.

After their date, Scottie escorts Judy back to her apartment. The neon sign outside her apartment window casts a green glow around her room. Hitchcock hides Judy within the shadows while Scottie asks to see more of Judy, an ironic juxtaposition of dialogue and imagery. Judy, suspecting that Scottie's affections are only for Madeleine, asks, “Why? Because I remind you of her?” (01:44:22). Judy is completely in silhouette, but, again, acts as a reflection of Madeleine. When first viewing Madeleine at Ernie's, Scottie and the audience receive a close-up, side profile shot of Madeleine's right side (Fig. 9). Here in her apartment, Judy wants to be seen for who she truly is but, instead, is concealed in the shadows with her left side profile facing Scottie and the audience (Fig. 10). These two profile shots symbolize the two sides to Judy/Madeleine. Hitchcock utilizes the darkness to shadow Judy in her apartment, signifying

Scottie's inability to see Judy because he is so deeply enamored by Madeleine's memory.

Sadly, Scottie's inability to let go of Madeleine produces a manic desire within him to transform Judy into Madeleine. In mathematics, transformations can be discussed in relation to functions. Specific additions or subtractions to a function can produce dramatic physical effects within its graph. The same transformation is forced upon Judy. Scottie demands clothing for Judy that Madeleine wore, and he demands her hair be not only dyed but also styled in the same spiralling updo. He does not love Judy and does not desire to do so; instead, Scottie insists Judy transform into Madeleine, becoming the perfect reflection of the love he lost. Having fallen in love with Scottie when she played Madeleine, Judy allows herself to be molded and manipulated back into Madeleine. Judy defeatedly asks Scottie, "If I let you change me, will that do it? If I do what you tell me, will you love me?" (01:51:05). Scottie longs for Madeleine to love; Judy longs for Scottie's love. The interdependence between the two characters mimics relations between dependent functions in mathematics; by changing one, the other will also be affected.

The green halo of light around Judy reminds the viewer of her seemingly apparent eternity as she continues to transform, reflect, and return repeatedly.

Judy's full transformation into Madeleine signals her final rebirth. Hitchcock again uses fractured close-up shots, mirroring the title sequence, of Judy's hair, lips, and nails during this transformation to communicate her fractured self. Judy cannot be her own full person but must, instead, take on the parts and roles of another, namely Madeleine, to be loved and accepted by Scottie. After returning from the beautician, Scottie is dissatisfied as Judy kept her original hairstyle and only changed the colour. This act of keeping the hairstyle indicates Judy's last stand. She desires so much to be loved as herself and wishes to retain some remnant of who she truly is. Scottie, desiring only Madeleine, commands Judy to style her hair in the same way Madeleine did. As Judy reenters the room after fixing her hair, her outline is blurred by the eerie neon green light from the hotel sign outside her window (01:55:30). She appears like a ghost, returning from the grave. After over a year of mourning, Scottie's love has finally resurfaced. The green halo of light around Judy reminds the viewer of her seemingly apparent eternity as she continues to transform, reflect, and return repeatedly. Scottie embraces Judy fully for the first time following her complete transformation. Before accepting her inevitable role as Madeleine and donning Madeleine's clothing and hairstyle again, Scottie did not engage in physical contact with Judy outside of dancing. Subconsciously, he could not be intimate or have physical affections with another woman; his heart belonged too completely to Madeleine and her memory.

The distance between Judy and Scottie is accentuated in two small scenes. The first is while they are walking along the water, and Judy glances down at a kissing couple. She longs

for that intimacy with Scottie. The second scene follows when Scottie purchases her new clothing. She is upset at being rejected for being herself and asks to be released from the relationship. Scottie tries to reassure her that he has feelings for her by grasping her face in his hands and saying, "No, Judy. It's you too. There's something in you..." (01:50:01). Scottie recognizes the falsity in his statement as he's talking and purposely removes his hands from her face. His hesitancy in making physical contact with Judy prior to her transformation makes the embrace following her transformation brutally heart wrenching.

The golden spiral reappears during Scottie's first full embrace of Judy post-transformation. He has embraced no one since Madeleine's death, and by hugging and kissing Judy, he is reminded of her prior reflection, that is to say Madeleine. Hitchcock uses the spiralling camera movement to signify a returning to what once was. Scottie's sins—his failure to act and protect Madeleine—are figuratively being erased. The clock is rewinding and the spiralling motion of the camera acts as a physical representation of that. Additionally, the background shifts from Judy's apartment to the stables at the Spanish Mission,

the last location Scottie embraced Madeleine. The continual spiralling motion of the camera also allows for the spiral of Judy's hair to be viewed. Carlotta's existence is still present and noted through this spiral symbol. Judy cannot escape the choices she made and now must remain a reflection of the woman she helped murder in order to receive the love she so desperately longs for.

Hitchcock continues utilizing symmetry and duality to highlight Carlotta's presence through Judy's duplicated spiralling updo, and also through her necklace. Previously in the film, while Madeleine is viewing Carlotta's portrait, the camera deliberately zooms in on Carlotta's necklace. Madeleine is not wearing any such necklace in any shots. Hitchcock is leaving a clue for the audience, something he wants them to pay attention to as the story evolves. As Judy prepares for a date with Scottie following her full transformation, she requests his assistance with a necklace. Importantly, Scottie first sees the necklace in Judy's mirror, through a reflection. Reflections in mathematics accentuate the duality between two objects across some line of symmetry. Here, Hitchcock's use of the mirror doubles down on the mirroring aspect between Carlotta, Madeleine, and Judy. A close-up shot of the necklace on Judy cuts to the painted image within Carlotta's portrait before fading back to Scottie's face. Scottie has finally realized the lies he has fallen for. Ironically, following his revelation, Judy kisses Scottie declaring, "Oh, Scottie. I do have you now, don't I?" (01:58:58). Little does Judy know that the veil of deception has been ripped asunder through her shortsighted selection of jewelry.

Hitchcock uses the Spanish Mission as a poignant symbol

of mastery for Scottie and as a unifier for Carlotta, Madeleine, and Judy. After realizing the truth about Judy, Scottie drives them out to the mission. The tension in the car is high while the suspense reaches peak levels for the audience when Scottie tells Judy, “One final thing I have to do and then I’ll be free of the past” (02:00:05). The juxtaposition between the two visits to the mission is startling. With Madeleine, bright sunshine bathes the two lovers while Scottie strives to release her from her inner demons and Carlotta’s control. With Judy, dark ominous clouds hang over the shadowed mission, symbolizing the turbulent scene to come. The weather and atmosphere around the mission also suggest how knowing the truth doesn’t always produce peace. Scottie’s heart is troubled with the truth he has discovered, and he wishes to free himself from his past through revisiting the mission.

Duality is at the forefront of the final sequence of *Vertigo*. Scottie has been to the mission before with Madeleine and is now there with Judy. He has raced up the hauntingly spiral staircase before, but where he once was frozen in fear, he is now motivated by anger. Scottie’s vertigo is ever-present as experienced by Hitchcock’s spiralling staircase technique, but Scottie redirects his attention away from his fear and toward understanding his manipulation. Love for Madeleine has morphed into anger at Elster and Judy. Scottie has been repetitively tricked, and the sense of injustice and anger fuels him to overcome his vertigo

and ascend the mission bell tower.

Sadly, the duality continues. After being spooked by an investigative nun, Judy plummets to her death from the same window through which Madeleine’s body was tossed. In a sense, Scottie has lost two of his loves in the same manner. Hitchcock continues his motif of rebirth and renewal through this final scene. Scottie’s final shot in the film shows him peering over the edge of the bell tower at Judy’s body. His outline is a reflection of his falling silhouette from his previous dream. Hitchcock uses the dream to foreshadow the scheme Scottie would fall into, but it is only by following Judy’s accidental demise that he fully understands the spiralling chaotic plan he was duped into believing.

Vertigo is a tragic story of duplicity, deception, disintegration, and demise. Mathematical concepts, such as reflection, the golden spiral, transformations, and colour theory riddle the captivating plot, offering deeper layers of manipulation and understanding. Carlotta is transformed into Madeleine who is transformed into Judy. Scottie, due to the tragic losses he experiences, also transforms from a fearful yet trusting man into a depressed manic shell of a man who is trapped by his own past. Alfred Hitchcock has utilized mathematics to heighten suspense, symbolism, and tension within *Vertigo* while ensnaring Scottie in his own repeating and spiralling chaotic life. ■

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